Week 29 – SA revision, No CAS (60 marks, 90 minutes + 10 min reading time)
Formula sheet allowed, no other notes or calculators allowed.

1. a) Let $y = \left(\frac{x^2}{3} - 2x\right)^5$, find $\frac{dy}{dx}$. (1)
   b) If $f(x) = \frac{x^2}{\cos(x)}$, find $f'\left(\frac{\pi}{3}\right)$. (2)

2. For a function $f: [-1, 3] \to R, f(x) = \frac{x^2}{e^x}$. Find its absolute maximum and minimum. (3).

3. Jacob is playing table tennis with 4 friends. The probability of winning a certain number of games is a random variable with a probability distribution given in the table below:

x	0	1	2	3	4
$\Pr(X = x)$	p^2	$\frac{4p^2}{3}$	$\frac{3p}{2}$	$\frac{p}{2}$	$\frac{2p^2}{3}$

   a) Find the value of p. (2)
   b) Find the probability that Jacob will win at least 1 game. (1)
   c) Jacob won the first two games. Find the probability that he will win at least 3 games in total. (2)

4. Elite soccer players were given several attempts to produce their longest kick. The length of their best kicks were normally distributed with a mean of 50m, and a standard deviation of 5m. It is known that $\Pr(Z < 1.4) = 0.75$.
   a) Find the probability that a randomly selected player will have a longest kick of at least 43m. (2)
   b) If a player was randomly chosen from those who were able to kick a ball further than 50m, what is the probability the player can kick further than 57m? (2)

5. Consider the following linear simultaneous equations:
   $3x - (k+2)y = k+1$
   $kx - 5y = 4$, where k is a constant.

   a) Find the value(s) of k for which there is a unique solution (2)
   b) Find the value(s) of k for which there is an infinite number of solutions (1)

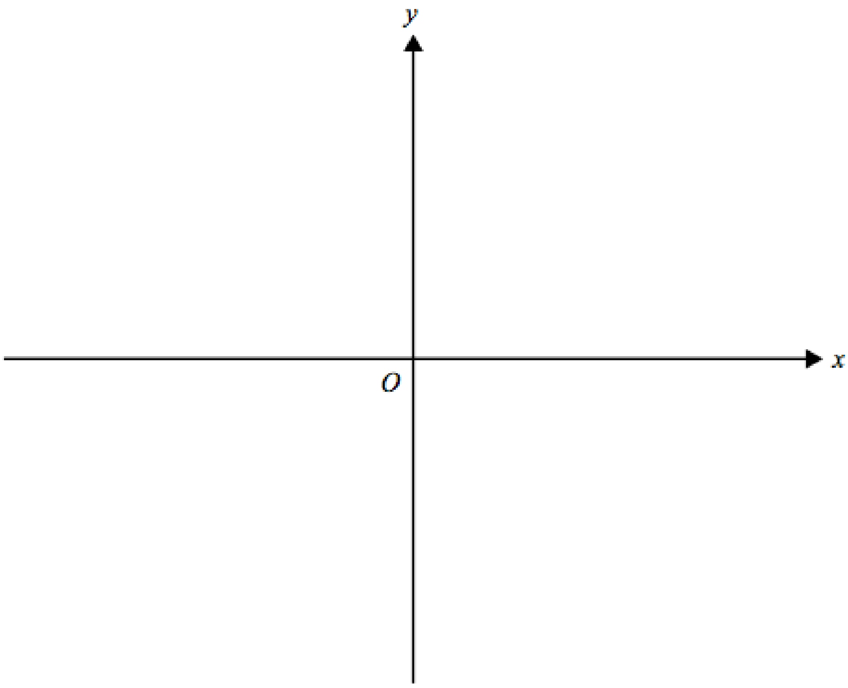
6. Find the general solution to the equation $\sqrt{3}\cos(2x) + \sin(2x) = 0$ (2)

7. A binomial distribution of the random variable X, with three independent trials, is such that $\Pr(X = 1) = \frac{p}{3}$, where p is the probability of a success on any trial and also 0 < p < 1. Determine the value of p. (2)

8. The average value of the function $f: (-\infty, 5) \to R, f(x) = \frac{1}{5-x}$ over [1, k] is $\frac{1}{2}\ln(2)$. Find the value of k. (3)

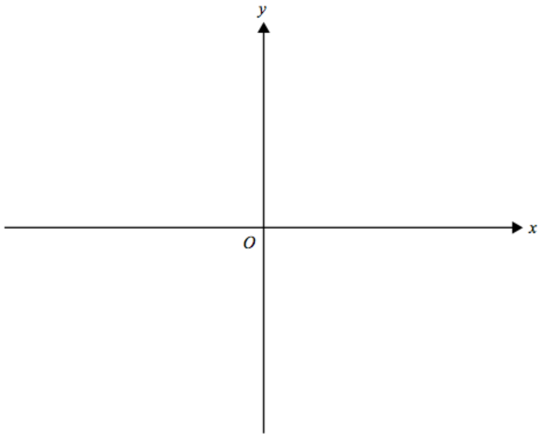
9. A transformation is defined by the matrix $\begin{bmatrix} 0 & 4 \\ -3 & 0 \end{bmatrix}$. Find the equation of the image of the graph of the line with the equation 2y – 3x = 5 under this transformation. (2)

10. a) For $f: [0,2\pi] \rightarrow R, f(x) = -2\sin(2x) - 1$, state the range and period of the function. (2)
b) Sketch the graph of f on the axes below. Label axes intercepts and endpoints with co-ordinates. (3)



c) $-2\sin(2x) - 1 = p$ has four solutions over the domain $[0,2\pi]$. State the intervals of p for which this will occur (1).

11. Consider the function $f: (-\infty, 4) \rightarrow R, f(x) = 3\log_e(4 - x)$.
a) Find the rule for the inverse function, $f^{-1}(x)$. (2)
b) On the axes below, sketch the graph of $f^{-1}(f(x))$. (1)



12. a) Solve $\log_6(3) - 2\log_6(x) + \log_6(2) = 1$ for x. (2)
b) Solve $8^{1-2x} = 2^{4+x}$ for x. (2)

13. Solve $\sin\left(\frac{x}{3}\right) = \frac{1}{\sqrt{2}}$ for $x \in [0,6\pi]$. (2)

14. For a discrete random variable X, with its probability distribution shown below:

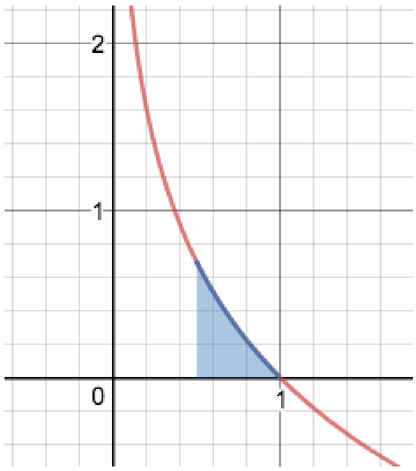
x	0	1	2	3	4
$\Pr(X = x)$	0.2	0.2	0.1	0.4	0.1

a) Find the expected value of X. (1)
b) Find $\Pr(X \leq 2 | X > 0)$. (2)
c) Find the variance of X, Var(X). (2)

15. For a continuous random variable X, its probability density function is given by:

$$f(x) = \begin{cases} a\cos(\frac{\pi x}{2}), x \in [0,1] \\ 0, elsewhere \end{cases}$$ (where a is a constant).

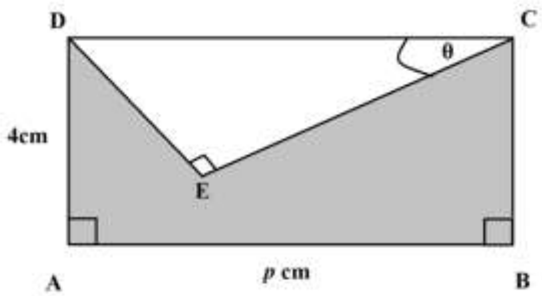
Find the value of a. (3)



16. Let $g: (0, \infty) \to R, g(x) = -\log_e(x)$. It is graphed to the right.

a) Differentiate $(1 - x)\log_e(x)$ with respect to x. (1)
b) Hence find the area of the shaded region. Express your answer in the form $a(\log_e(a) + 1)$, where a is a positive, real constant (3)

17. In the diagram below, ABCD is a rectangle, and CDE is a right-angled triangle where $\angle DCE = \theta$ and $0 < \theta < \frac{\pi}{2}$. Also, AD = BC = 4cm, and AB = p cm, where p is a positive constant.



a) Find an expression in terms of p and θ for DE and CE (2).
b) The area of the shaded region in the diagram is $A\text{cm}^2$. Find an expression for A in terms of p and θ. (1)
c) Find the value of θ to make A is a minimum. (2)
d) If the minimum area of the shaded region is 16cm^2, find the value of p. (1)

Week 30 – MCQ revision, CAS (60 marks, 90 minutes + 10 min reading time)
Any type of written material allowed.

Question 1
One root of the equation $x^2 + kx + k = 0$ is $x = -\frac{1}{2}$. The other root is:

A. $-\frac{7}{8}$
B. $-\frac{1}{2}$
C. 1
D. $\frac{7}{8}$
E. 2k

Question 2
The number of solutions there are to the equation $\sin(x) = \sin^2(x)$ if $x \in [0, 2\pi]$ is:

A. 2
B. 3
C. 4
D. 5
E. 6

Question 3
Given $f: [-a, 3a) \to R, f(x) = 2a - x$ where $a \in R\backslash\{0\}$, then the range of the function is:

A. $[-a, 3a)$
B. $(-a, 3a]$
C. $(-a, 3a)$
D. $[-a, 3a]$
E. $(3a, -a]$

Question 4
Given $f(x) = a\tan\left(\frac{b\pi x}{3}\right)$ where $a, b \in R$, has period equal to:

A. $\frac{6}{b}$
B. $\frac{3}{b}$
C. $\frac{3\pi}{b}$
D. $\frac{2\pi b}{3}$
E. $\frac{\pi ab}{3}$

Question 5
The transformation $T: R^2 \to R^2$, which maps the curve with equation $y = \log_e(x)$ to the curve with equation $y = 2 - 2\log_e(2x + 2)$, has the rule:

A. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix}$
B. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix}$
C. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \end{bmatrix}$
D. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -2 \\ -2 \end{bmatrix}$

E. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

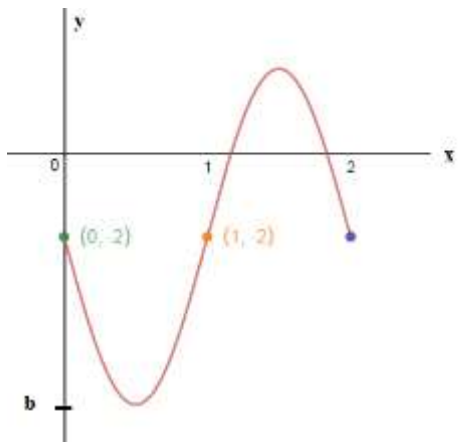
Question 6

The function $f: [0, 2\pi] \to R, f(x) = 2 - \sin\left(\frac{\pi}{3} - 2x\right)$ has amplitude and period of:

A. 3 and 2
B. 3 and 2π
C. 1 and π
D. 2 and π
E. 1 and $\frac{\pi}{2}$

Question 7

The diagram below shows one cycle of the graph of a circular function, where b < -2. A possible equation for the function whose graph is shown is:



A. $y = -(b-2)\sin(2x) - 2$
B. $y = 2 - b sin(2x)$
C. $y = 1 - (b+2)\sin(2\pi x)$
D. $y = (b+2)\sin(\pi x) - 2$
E. $y = (-b-2)\sin(2x) + 2$

Question 8

One cycle of the graph of the function with the equation $y = \tan(ax)$ has successive vertical asymptotes at $x = \frac{1}{12}$ and $x = \frac{1}{4}$. A possible value of a is:

A. 12
B. 12π
C. 4
D. 4π
E. 6π

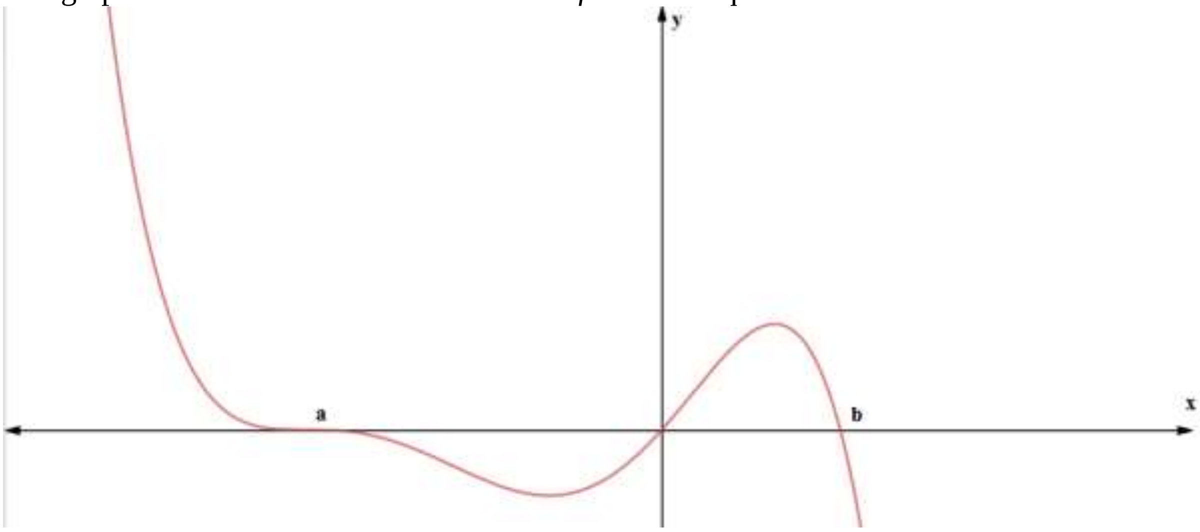
Question 9

The transformation $T: R^2 \to R^2$, which maps the curve with equation $y = e^x$ to the curve with equation $y = e^{2x+4} - 3$ has the rule:

A. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -2 \\ -3 \end{bmatrix}$

B. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -2 \\ -3 \end{bmatrix}$
C. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 4 \\ -3 \end{bmatrix}$
D. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -4 \\ 3 \end{bmatrix}$
E. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 4 \\ 3 \end{bmatrix}$

Question 10
The graph shown could be that of a function *f* with the equation:



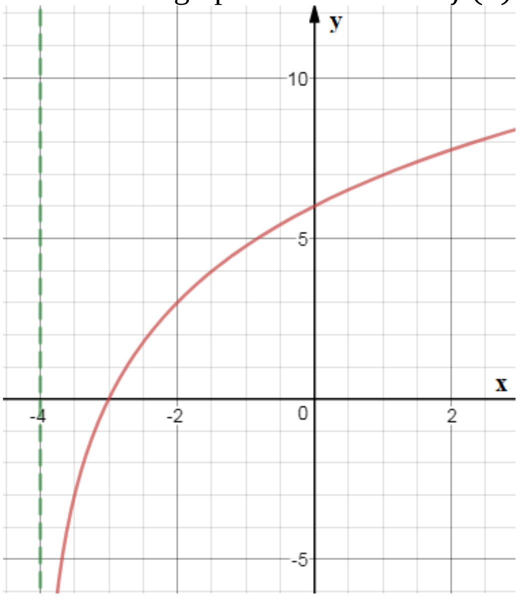
A. $y = -x(x+a)^3(x-b)$
B. $y = -x(x+a)^2(x-b)$
C. $y = x(x-a)^3(x-b)$
D. $y = -x(x+a)^3(b-x)$
E. $y = x(x-a)^3(b-x)$

Question 11
If $f: [-2, \infty) \to R$ and $g: [-5,5] \to R$ then the graph of h(x) where h(x) = f(x) + g(x) will have a domain of:
A. $[-5, -2]$
B. $[-5, \infty)$
C. $[-2, 5)$
D. $[-2, 5]$
E. $(-\infty, -2]$

Question 12
The equations of the asymptotes of $y = -2 - \frac{1}{(9+3x)^2}$ are:
A. x = -9, y = -2
B. x = 3, y = -2
C. x = 3, y = 2
D. x = -3, y = -2
E. x = $-\frac{1}{3}$, y = -2

Question 13

Consider the graph of the function $f(x) = alog_e(x + b)$, shown below:



A set of possible values of a and b is:

A. a = 6, b = 4
B. a = $6\log_e 4$, b = 4
C. a = $\frac{6}{\log_e 4}$, b = 4
D. a = $\frac{4}{\log_e 6}$, b = -4
E. a = $6e^4$ b = 4

Question 14

The midpoint of the line segment that joins (1, -5) to (d, 2) is

A. $\left(\frac{d+1}{2}, -\frac{3}{2}\right)$
B. $\left(\frac{1-d}{2}, -\frac{7}{2}\right)$
C. $\left(\frac{d-4}{2}, 0\right)$
D. $\left(0, \frac{1-d}{3}\right)$
E. $\left(\frac{5+d}{2}, 2\right)$

Question 15

If $x + a$ is a factor of $7x^3 + 9x^2 - 5ax$, where $a \in R\backslash\{0\}$, then the value of a is

A. -4
B. -2
C. -1
D. 1
E. 2

Question 16

Let $g: R \rightarrow R, g(x) = ax^4 - bx^2$ where a and b are positive real numbers. Also, g'(r) = 0 and g'(s) = 0 where r and s are real numbers and r < s. The function g has a positive gradient for:

A. $x \in (-\infty, s) \cup (0, r)$

B. $x \in (-\infty, r) \cup (0, s)$
C. $x \in (r, s)$
D. $x \in (s, 0) \cup (r, \infty)$
E. $x \in (r, 0) \cup (s, \infty)$

Question 17
A cubic has stationary points such that h'(p) = 0 and h'(q) = 0. The graph of h passes through the points (p, 2) and (q, 5). The equation h(x) + c = 0 has three solutions if:
A. $c > -5$
B. $c < -2$
C. $-5 < c < -2$
D. $2 < c < 5$
E. $c > -2 \; and \; c < -5$

Question 18
Consider the following set of functions, each defined over its maximal domains. $f(x) = x, g(x) = \frac{4}{x}, h(x) = \frac{x}{x-1}, j(x) = \frac{x-2}{x}$. Which of these functions has the property that its inverse is identical to itself?
A. f only
B. f and g only
C. f, g and h only
D. All of the functions
E. None of the functions

Question 19
Let $f(x) = \sqrt{x+9}$ and $g(x) = x - 6$. The domain of $\frac{f}{g}(x)$ equals:
A. $(-6, 9]$
B. $[-9, \infty)$
C. $[-9,6)$
D. $[-9,6) \cup (6, \infty)$
E. $(-\infty, -6) \cup (-6,9]$

Question 20
The function $f: (-\infty, a] \rightarrow R, f(x) = -x^3 + 4x^2 + 3x - 2$ will have an inverse provided
A. $a \leq -\frac{1}{3}$
B. $a \geq -\frac{1}{3}$
C. $a \leq 3$
D. $a \in \left[-\frac{1}{3}, 3\right]$
E. $a \leq 0$

Question 21
The cubic function $f: R \rightarrow R, f(x) = ax^3 + bx^2 + cx + d$,where a, b, c, d $\in R \backslash \{0\}$ will have no turning points when
A. $b^2 > 3ac$
B. $b^2 \leq 3ac$
C. $b^2 < 4ac$
D. $b^2 > 4ac$

E. $b^2 = 4ac$

Question 22
Let $f(x) = x^3$. If a and b are non-zero real numbers, which of the following is false?

A. $f(ab) = f(a)f(b)$
B. $\frac{f(a)}{f(b)} = f\left(\frac{a}{b}\right)$
C. $f(b) + f(-b) = 0$
D. $f(2a) = 8f(a)$
E. $f(a+b) = f(a) + f(b)$

Question 23
If $f(x) = e^x$ and $g(x) = \frac{1}{\sqrt{2x}}$, then $g(f(0))$ is:

A. $\frac{1}{2e}$
B. undefined
C. $\frac{1}{2\sqrt{e}}$
D. $\frac{1}{\sqrt{2}}$
E. $\frac{1}{\sqrt{2e}}$

Question 24
The inverse function of the function $f: [0,5] \to R, f(x) = -\sqrt{25 - x^2}$ is:

A. $f^{-1}: (5, \infty) \to R, f^{-1}(x) = \sqrt{x^2 - 25}$
B. $f^{-1}: [-5,0] \to R, f^{-1}(x) = \sqrt{25 - x^2}$
C. $f^{-1}: [0,5] \to R, f^{-1}(x) = \sqrt{25 - x^2}$
D. $f^{-1}: (-5,0] \to R, f^{-1}(x) = \sqrt{25 - x^2}$
E. $f^{-1}: [0,5] \to R, f^{-1}(x) = -\sqrt{25 - x^2}$

Question 25
The value(s) of k for which the following system of equations have no solution is/are:

$kx + 2y = 4$
$x + (k-1)y = 2$, where k is a constant.

A. $k = -1, 2$
B. $k = -1$
C. $k = 2$
D. $k \neq 2$
E. $k \neq -1$

Question 26
If $f(x) = x^2 - 1, x \geq 1$ and $g(x) = \sqrt{2 + 2x}, x \geq -1$ then $g(f(x))$ is defined as:

A. $g\big(f(x)\big) = \sqrt{2}x, x \geq -1$
B. $g\big(f(x)\big) = -\sqrt{2}x, x \geq -1$
C. $g\big(f(x)\big) = -\sqrt{2}x, x \geq 1$
D. $g\big(f(x)\big) = \sqrt{2}x, x \geq 1$
E. $g\big(f(x)\big) = \sqrt{2}x, x \geq 0$

Question 27
The inverse of $f(x) = 2 - 3\log_e(1-x)$ is given by $f^{-1}(x) = a - e^{bx+c}$. The values of a, b and c are:

A. $a = 1, b = -1, c = 2$
B. $a = 1, b = \frac{2}{3}, c = -\frac{1}{3}$
C. $a = 1, b = -\frac{1}{3}, c = \frac{2}{3}$
D. $a = -1, b = -\frac{1}{3}, c = \frac{2}{3}$
E. $a = 1, b = \frac{1}{3}, c = -\frac{2}{3}$

Question 28
If the function $f(x) = x^3 - 2ax + b$ has a stationary point at (2, 5), the values of a and b respectively are:

A. a = 6, b = -21
B. a = 0, b = 21
C. a = -6, b = -27
D. a = 6, b = 21
E. a = 3, b = -10

Question 29
The function h can be differentiated for all real values of x. The derivative of the function $h(\cos(2x))$ is given by:

A. $h'(\cos(2x))$
B. $h'(-2\sin(2x))$
C. $-2h'(\cos(2x))\sin(2x)$
D. $2\sin(2x)h'(\cos(2x))$
E. $2\sin(2x)\,h(\cos(2x))$

Question 30
Let $h: R \to R, h(x) = x^2e^x$. The average value of h over [0, r] is $\frac{5e^4-1}{2}$. The value of r is

A. 3
B. 3.178
C. 4
D. 4.522
E. 5

Question 31
Given that $\int_1^3 g(x)dx = -2$, then $\int_1^3 (4x + 2g(x))dx$ is equal to:

A. 4
B. 12
C. 18
D. 20
E. 30

Question 32
The graph of $y = e^x$ has a tangent at the point (p, e^p). This tangent crosses the y-axis at (0, q). Given that $q < 0$, then all the possible values of p are given by:

A. p < -e
B. p > 1/e
C. p > 1
D. p > 3
E. 1 < p < e

Question 33
Refer to the following table:

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
0	3	0	-4	3
1	5	2	-1	-3
2	1	5	6	-2
3	7	9	12	-1

If h(x) = g(f(x)), then the value of $h'(2)$ equals
A. -18
B. -12
C. -3
D. 2
E. 12

Question 34
The area bounded by the line y = x and the graph of the parabola y = x^2 – x is cut in half by a line with equation x = k. What is the value of k?
A. 1/2
B. 3/4
C. 1
D. 5/4
E. 5/3

Question 35
For the function with the rule $f(x) = \sqrt{b^2 - x^2}, b \in R^+$. The average rate of change of f(x) with respect to x on the interval [0, b/2] is:
A. $b(\sqrt{3} - 2)$
B. $\sqrt{3} - 2$
C. $-\frac{\sqrt{3}}{3}$
D. $-\frac{b\sqrt{3}}{3}$
E. $\frac{b}{2}(\sqrt{3} - 2)$

Question 36
Consider $f: R \to R, f(x) = ax^2 - 2bx$, where $a, b \in R\backslash\{0\}$. Then
A. If a > 0 and b > 0, the function is strictly decreasing for 0 < x < 2b
B. If a > 0 and b < 0, the function is strictly decreasing for x > 0
C. If a > 0 and b > 0, the function is strictly decreasing for x > b/a
D. If a < 0 and b > 0, the function is strictly decreasing for x < b/a
E. If a < 0 and b > 0, the function is strictly decreasing for x > b/a

Question 37

The velocity v(t) of a moving particle at a time *t* seconds is given by $v(t) = \frac{27}{(3t+4)^2} m/s$ for $t \geq 0$. Initially the particle is at rest at the origin. Which of the following is false?

A. The initial velocity is 27/16 m/s.
B. The distance travelled in the first two seconds is 27/20 metres.
C. The acceleration of the particle is given by $a(t) = \frac{-162}{(3t+4)^3} m/s^2$
D. The distance travelled at a time *t* is equal to $\frac{-9}{3t+4}$ metres.
E. The position is given by $x(t) = \frac{27t}{4(3t+4)}$ metres.

Question 38

If $y = \log_e\left(\sqrt{f(x)}\right)$ then $\frac{dy}{dx}$ is equal to:

A. $\frac{1}{\sqrt{f(x)}}$
B. $\frac{1}{2\sqrt{f(x)}}$
C. $\frac{f'(x)}{2\sqrt{f(x)}}$
D. $\frac{f'(x)}{2f(x)}$
E. $\frac{1}{2f(x)}$

Question 39

The average value of $y = 2e(e^x - 1)$ over the interval [0,2] is:

A. $2e^3 - 6e$
B. 0
C. $\frac{\left(e^2-1\right)^2}{2}$
D. $e^3 - 3e$
E. $(e^2 - 1)^2$

Question 40

If $f'(x) = \frac{5}{\cos^2(kx)}$ and *k* and *c* are real constants, then $f(x)$ is equal to:

A. 5tan(kx) + c
B. 5k tan(x) + c
C. $5\sin^2(kx) + c$
D. 5/k tan(kx) + c
E. $5\log_e(\cos(kx)) + c$

Question 41

Let *p* be a function defined for the interval [a, b] such that $q'(x) = p(x)$, for all $x \in [a, b]$. Hence, $\int_a^b p(2x)dx$ is equal to:

A. $q(2x) + c$
B. $p(2b) - p(2a)$
C. $\frac{1}{2}\left(q(2b) - q(2a)\right)$
D. $\frac{1}{2}\left(q'(2b) - q'(2a)\right)$
E. $q(2b) - q(2a)$

Question 42

The line $y = 3x$ is a tangent to the curve with equation $y = \frac{-2k}{x-1}$ when:

A. $k \in R$
B. $-\frac{9}{24} < k < \frac{9}{24}$
C. $k \leq \frac{9}{24}$
D. $k \geq \frac{9}{24}$
E. $k = \frac{9}{24}$

Question 43

If $f'(x) = 2\sin\left(\frac{5x}{2}\right)$ then $f(x)$ could be:

A. $-\frac{4}{5}\cos\left(\frac{5x}{2}\right)$
B. $-\frac{1}{5}\cos\left(\frac{5x}{2}\right) - 9$
C. $\frac{4}{5}\cos\left(\frac{5x}{2}\right) + 5$
D. $\frac{1}{5}\cos\left(\frac{5x}{2}\right) + 7$
E. $\frac{4}{5}\sin\left(\frac{5x}{2}\right) + 5$

Question 44

The number of bacteria, N, in a colony varies with time according to the rule $N = N_0 e^{0.1t}$, where *t* is the time measured in days, and t ≥ 0. If initially there were 1000 bacteria, then the average rate of change in the number of bacteria over the first 10 days is closest to:

A. 172
B. 183
C. 272
D. 1718
E. 2718

Question 45

The continuous random variable X has a probability density function given by:

$$f(x) = \begin{cases} \log_e(x+1), & 0 < x < e-1 \\ 0, & elsewhere \end{cases}$$

Pr(X<1) is equal to:

A. $2 - 2\log_e(2)$
B. $2\log_e(2) - 1$
C. $2 - \log_e(2)$
D. $(e+1)\log_e(e+1) - e$
E. $(e+1)\log_e(e+1) - 2\log_e(2) - e + 1$

Question 46

The heights of a large group of army recruits are normally distributed with a mean of 179cm and a standard deviation of 5cm. The tallest 20% of these recruits are invited to trial for a particular unit. The minimum height, in cm, required to trial for this unit is closest to

A. 174.8
B. 179.6

C. 183.2
D. 189.0
E. 194.0

Question 47
Kevin is a quality control officer on a production line. He is checking car components for faults. The probability that he finds a faulty component is p, where p < ½. The probability of one component being fault is independent of the probability of the next component being faulty. The probability that three of the next four components he checks are fault is 0.1536. The value of p is:

A. 0.16
B. 0.3
C. 0.36
D. 0.4
E. 0.47

Question 48
In a sample space containing the events A and B, Pr(A) = p where 0 < p < 1. Also, $\Pr(A|B) = 0$ and $\Pr(A' \cap B') = 2p$. Pr(B) is equal to:

A. p
B. 3p
C. 1 – p
D. 1 – 3p
E. $1 - 2p^2$

Question 49
A tennis player gets her serve in play 65% of the time. The day prior to the tournament, this player practises by having 180 first serves, one at a time. Assuming the outcome of any one serve is independent of another, the mean and standard deviation of the number of successful first serves is:

A. $\mu = 117$ and $\sigma = \frac{3\sqrt{455}}{10}$
B. $\mu = 117$ and $\sigma = \frac{2\sqrt{35}}{13}$
C. $\mu = 63$ and $\sigma = \frac{3\sqrt{455}}{10}$
D. $\mu = 63$ and $\sigma = \frac{2\sqrt{35}}{13}$
E. $\mu = 117$ and $\sigma = \frac{819}{20}$

Question 50
An insurance company insures a large number of shops against damage from vandals. The insured value, *V*, in units of $100,000, of a randomly selected shop is assumed to follow a probability distribution with density function: $f(v) = \begin{cases} \frac{3}{v^4}, v > 1 \\ 0, \ elsewhere \end{cases}$
Given that a randomly selected shop is insured for at least $150,000, the probability that it is insured for under $200,000 is:

A. 0.578
B. 0.684
C. 0.704

D. 0.829
E. 0.875

Question 51

Given the continuous probability distribution is defined by $f(x) = \begin{cases} ksin(3x), 0 \leq x \leq \frac{\pi}{3} \\ 0, \ elsewhere \end{cases}$

Which of the following is false?

A. $k = \frac{3}{2}$
B. The median is equal to $\frac{\pi}{6}$
C. $E(X) = \frac{\pi}{6}$
D. $E(X^2) = \frac{\pi^2}{36}$
E. $\Pr\left(0 < X < \frac{\pi}{4}\right) = \frac{2+\sqrt{2}}{4}$

Question 52

The random variable X has the following probability distribution.

x	1	2	3
$\Pr(X = x)$	a	$\frac{a}{2}$	$\frac{a}{3}$

Which of the following statements is false?

A. $a = \frac{6}{11}$
B. $E(X) = \frac{18}{11}$
C. $E(X^2) = \frac{36}{11}$
D. $Var(X) = \frac{72}{121}$
E. $E\left(\frac{1}{X}\right) = \frac{11}{18}$

Question 53

Orange Juice is packed in small glass bottles containing 175 mL. The packing process produces bottles that are normally distributed with a mean of 176 mL. In order to guarantee that only 1% of bottles are under-volume, the standard deviation for the volume, in ML, would be required to be closest to:

A. 0.01
B. 0.99
C. 0.43
D. 176.01
E. 175.01

Question 54

The number of defective batteries in a box of batteries ready for sale is a random variable having a binomial distribution with a mean of 12 and a variance of 10. If a battery is drawn at random from the box, the probability that it is not defective is:

A. 1/6
B. 5/6
C. 1/72
D. 1/12
E. 9/10

Question 55
The random variable *X* has the following probability distribution:

x	2	4	6
$\Pr(X = x)$	a	b	0.1

If the mean of *X* is 2.6, then the values of a and b, respectively, are

A. 0.5, 0.4
B. 0.8, 0.1
C. 0.1, 0.8
D. 0.3, 0.7
E. 0.4, 0.5

Question 56
A random variable *X* is normally distributed with mean 4.7 and standard deviation 1.2. If *Z* is the standard normal variable, then Pr(X < 2.3) is:

A. Pr (Z < 2)
B. Pr (Z < 1)
C. Pr (-2 < Z < 2)
D. Pr (Z > 2)
E. 1 – Pr (Z > 2)

Question 57
X ~ Bi(n, p) is a binomial random variable with mean 20 and standard deviation 4. The values of *n* and *p* respectively are:

A. 80 and 0.2
B. 80 and 0.8
C. 25 and 0.8
D. 16 and 0.2
E. 100 and 0.2

Question 58
For the following discrete probability distribution, the value of E(2X) is:

x	1	2	4	8
$\Pr(X = x)$	0.3	0.2	0.4	0.1

A. 3.1
B. 3.3
C. 6.2
D. 9.3
E. 13.9

Question 59
For events *A* and *B*, $\Pr(A \cap B) = p, \Pr(A' \cap B) = p - \frac{1}{8}$ and $\Pr(A \cap B') = \frac{3p}{5}$. If *A* and *B* are independent, then the value of p is:

A. 0
B. ¼
C. 3/8
D. ½
E. 3/5

Question 60

An opinion pollster reported that for a random sample of 574 voters in a town, 76% indicated a preference for retaining the current council.

An approximate 90% confidence interval for the proportion of the total voting population with a preference for retaining the current council can be found by evaluating

A. $\left(0.76 - \sqrt{\dfrac{0.76 \times 0.24}{574}},\ 0.76 + \sqrt{\dfrac{0.76 \times 0.24}{574}}\right)$

B. $\left(0.76 - 1.65\sqrt{\dfrac{0.76 \times 0.24}{574}},\ 0.76 + 1.65\sqrt{\dfrac{0.76 \times 0.24}{574}}\right)$

C. $\left(0.76 - 2.58\sqrt{\dfrac{0.76 \times 0.24}{574}},\ 0.76 + 2.58\sqrt{\dfrac{0.76 \times 0.24}{574}}\right)$

D. $\left(436 - 1.96\sqrt{0.76 \times 0.24 \times 574},\ 436 + 1.96\sqrt{0.76 \times 0.24 \times 574}\right)$

E. $\left(0.76 - 2\sqrt{0.76 \times 0.24 \times 574},\ 0.76 + 2\sqrt{0.76 \times 0.24 \times 574}\right)$

Week 31 – ER revision, CAS (60 marks, 90 minutes + 10 min reading time)

Any type of written material allowed.

Question 1

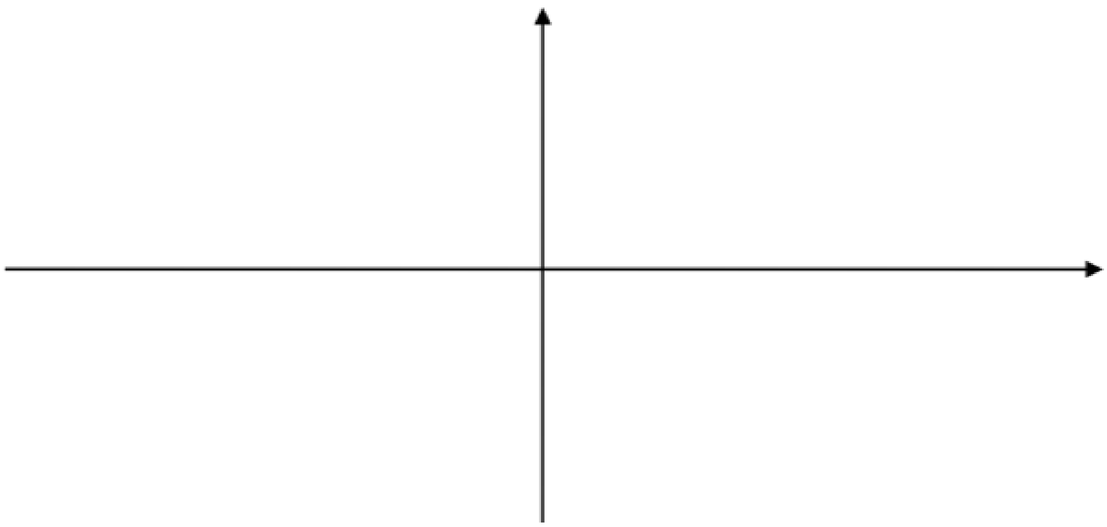
Consider the functions $f(x) = 2\log_e(x)$ and $g(x) = x + 1$

a) State the maximal domain and range of f and g. (1)

b) If we restrict the domain of g to (a, ∞), find the smallest value of a such that $f[g(x)]$ exists, and find a rule for $f[g(x)]$. (2)

c) For this value of a, sketch the function $f[g(x)]$ on the axes below. Label any asymptotes and intercepts. (2)

d) F



$(f \circ g)^{-1}(x)$ and state its domain and range. (2)

e) Evaluate $\int_0^{2\pi} (f \circ g)^{-1}(x)\, dx$, and shade an equivalent area on your graph from part c. (2)

f) Hence find $\int_0^{e^{\pi}-1} (f \circ g)(x) dx$ (3)

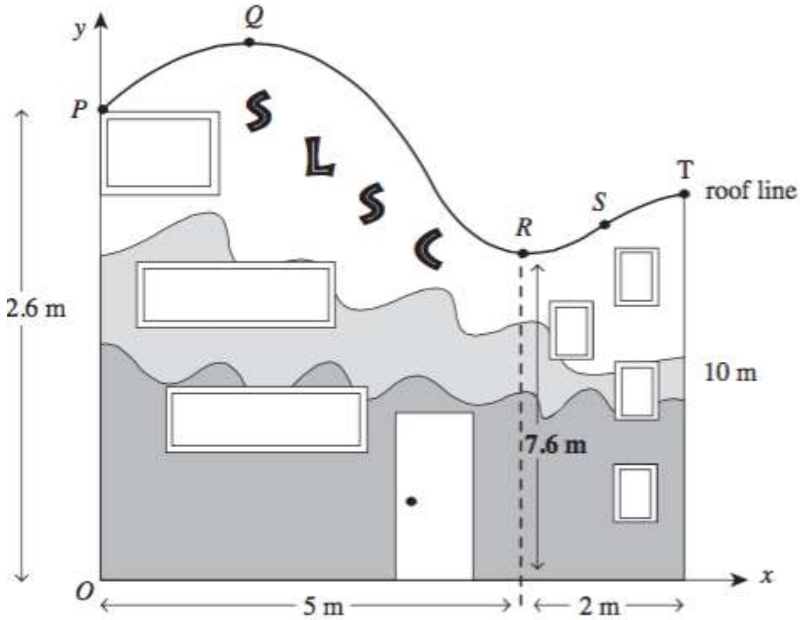
Question 2

Given the function $f: D \rightarrow R, g(x) = 3\log_e\left(1+\frac{x}{2}\right)$,

a) Find *D*, which is the maximal domain of the function *f*. (1)
b) Show that the graph of *f* has no turning points. (1)
c) State a sequence of transformations which takes the graph of $y = \log_e(x)$ to the graph of *f*. (2)
d) For $f(x) = 3\log_e\left(1+\frac{x}{2}\right)$:
   i. If $f(u-2) + f(v-2) = f(auv+b)$, where *u* and *v* are positive real numbers, find the values of *a* and *b*. (2)
   ii. For what values of *u* does $f(u) + f(-u) = f\left(-\frac{u^2}{2}\right)$ hold? (2)
e) Find the inverse function f^{-1}. (2)

Question 3

An architect is designing a new clubhouse for a local Surf Life Saving Club (SLSC). The roof is to be designed to look like a smooth wave. A diagram of the side profile of the clubhouse is shown below with key measurements added. Relative to the point *O*, the point *P* has coordinates (0, 12.6), the point *Q* is a local maximum, the point *R* is a local minimum and the roof is to be horizontal at the point *T*, whose coordinates are (7, 10).



The architect's first attempt to design a smooth roof is to use a hybrid function *f*, where f(x) metres measures the height of the roof above the ground and *x* metres measures the horizontal distance from *O*.

$$f(x) = \begin{cases} 0.2x^3 - 1.8x^2 + 3x + 12.6, x \in [0,6] \\ ax^2 + bx + c, x \in (6,7] \end{cases}$$

a) Verify the function *f* meets the requirements of the roofline at the point *R* as shown on the diagram above, i.e. show that it is (5, 7.6) and that it is a local minimum. (3)
b) Determine the y-coordinate of the point *S*, given that the x-coordinate at *S* is 6. (1)

c) The function *f* is continuous at *S*. Show that the rule for the section *ST* of the roof is given by $y = -x^2 + 14x - 39$. (3)
d) Show that the derivative function for the model is not defined at x = 6, i.e. the function is not smooth at the point *S*. (3)
e) The architect decides to modify the hybrid function by changing the parabolic part of the function to a cubic with a stationary point of inflection at the point *T*. Determine the equation of this cubic and whether or not this makes the model smooth at the point *S*. (3)

Question 4

A chocolate factory makes teddy bear statues. The weight of the statues is normally distributed with a mean of 1000 grams and standard deviation of 4 grams.

a) Find the probability, to four decimal places, a statue weights between 992 grams and 1010 grams. (1)
b) Statutes whose weight does not lie between 992 and 1010 grams are rejected as being underweight or overweight. If 1200 statues are manufactured over a particular period, find the number of statues, correct to the nearest whole number, likely to be rejected during this period of production. (1)
c) To reduce the number of overweight statutes to 10% of rejects, the machine can be modified to change the standard deviation while retaining the mean at 1000 grams. Find, to the nearest whole number, what the new standard deviation should be. (2)
d) Five statues are selected at random. What is the probability, correct to three decimal places, that at least one of the statutes is outside the desired weight of 992 grams to 1010 grams? Assume the standard deviation is 4. (2)

Each month the chocolate statutes are sold to a department store. There is an 85% chance the department store will buy statutes this month if it bought them last month. If the department store did not buy the statues last month, there is a 30% chance they will buy them this month.

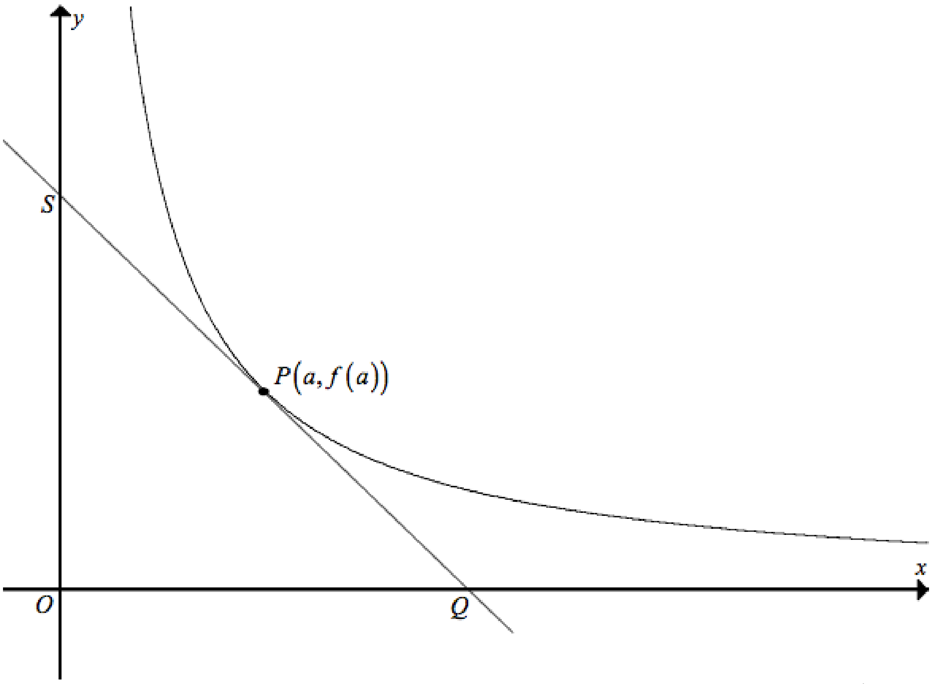
e) Suppose the department store bought statues last month.
    i. Find the probability, correct to four decimal places, that the department store will buy statues for the next three months. (1)
    ii. Find the probability, correct to four decimal places, that it buys statutes for two of the next three months. (2)
    iii. As a form of quality assurance, this department store takes samples of 20 chocolate statues at a time to test if a statue is underweight (with a standard deviation of 4). This is represented by the random variable $\hat{P}$. Find the standard deviation of $\hat{P}$. (1)

At a different department store, the probability it buys statues this month if it bought them last month is *p*. If it did not buy statues last month, the probability it will buy statues this month is *p – 0.1*. This department store bought statues in January and the probability it will buy in March is 0.7.

f) Show that $p = \frac{8}{11}$. (2)
g) What is the expected number of months, from February to March that the department store will buy statues? Give the answer correct to one decimal place. (3)

Question 5

The diagram below shows part of the graph of the function $f: (0, \infty) \to R, f(x) = \frac{1}{x}$.



a) Let $P(a, f(a))$ where a > 0 be a point on the graph of $y = \frac{1}{x}$.

i. Find, in terms of a, the equation of the tangent to the curve at point P. (2)

ii. The tangent to the curve at P crosses the x-axis at Q and the y-axis at S, as shown in the diagram above. Write down the coordinates of points Q and S. (2)

iii. Find the area of the triangle OQS and hence show that it is independent of a. (1)

b) Let n be a positive integer.

i. By considering the graph of $y = \frac{1}{x}$ show and explain why $\frac{1}{n+1} < \int_n^{n+1} \frac{1}{x} dx < \frac{1}{n}$ (2)

ii. Hence, show that $\left(1 + \frac{1}{n}\right)^n < e < \left(1 + \frac{1}{n}\right)^{n+1}$ (3)